5-2-31

2,31

$$y' = \frac{x^2 + 2xy - 5y^2}{2x^2 - 6xy}$$

Однородное уравнение, замена $y = tx$, $y' = t'x + t$

$$t'x + t = \frac{x^2 + 2tx^2 - 5t^2x^2}{2x^2 - 6tx^2}$$

$$t'x + t = \frac{1 + 2t - 5t^2}{2 - 6t}$$

$$t'x = \frac{1 + 2t - 5t^2}{2 - 6t} - t$$

$$t'x = \frac{1 + 2t - 5t^2 - 2t + 6t^2}{2 - 6t}$$

$$t'x = \frac{1 + t^2}{2 - 6t}$$

$$\frac{dt}{dx}x = \frac{1 + t^2}{2 - 6t}$$

$$\frac{2 - 6t}{1 + t^2}dt = \frac{dx}{x}$$

$$\int \frac{2 - 6t}{1 + t^2}dt = \int \frac{dx}{x}$$

$$\int \frac{2 - 6t}{1 + t^2}dt = 2\int \frac{1}{1 + t^2}dt - 6\int \frac{t}{1 + t^2}dt = 2arctg(t) - 3\int \frac{dt^2}{1 + t^2} =$$

$$= 2arctg(t) - 3\int \frac{d(t^2 + 1)}{1 + t^2} = 2arctg(t) - 3\ln(1 + t^2) + C$$

$$\int \frac{dx}{x} = \ln(x) + C$$

$$2arctg(t) - 3\ln(1 + t^2) = \ln(x) + C$$

$$C = 2arctg(t) - 3\ln(1 + t^2) - \ln(x)$$

Обратная замена

$$C = arctg\left(\frac{y}{x}\right) - 3\ln\left(1 + \frac{y^2}{x^2}\right) - \ln(x)$$